In: Mathematical Models of Non-linear Phenomena … ISBN: 978-1-60876-994-0
Editors: A. B. Nadykto, L. Uvarova et al.

Chapter 3

ON ONE CLASS OF EXACT AND APPROXIMATE SOLUTIONS OF EMDEN'S *E* – EQUATION FOR THE POTENTIAL

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ABSTRACT

During the integration of Emden's *E*-equation for the density of a gaseous sphere a mathematical problem of choice of boundary conditions emerges. The complete system of equations that is being solved with respect to the potential can be reduced to a three-dimensional equation of the same form. In this article it has been shown, that the correct choice of the boundary conditions in the equation for the potential can not be arbitrary when solving a spherical problem of self-consistent gravitation theory, but is prescribed by the first integral of total pressure that exists in a plane-symmetric system. Solutions obtained here describe the distribution of physical characteristics in astrophysical objects.

Keywords: Emden's E-equation, boundary conditions, self-consistent gravitation theory, integral of total pressure, astrophysical objects

1. INTRODUCTION

Emden *E*-equation, derived in [1], belongs to the class of ordinary second order differential equations. Together with Lane-Emden equation it played an important role at the first stage of finding solutions of problems of stellar structure. Stars have been considered as gaseous formations that stay either in polytropic equilibrium or in balance with uniform

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temperature distribution [2]. While solving the spherical problem of density distribution in a system with uniform temperature distribution, Emden could not find the analytic solution, which would reach a maximum density in the center of the sphere.

Independently of Emden's research and, seemingly, not knowing about it, Frencel in 1948 introduced the similar method of calculating the fields of gravitating particle for systems with the constant temperature, and named the macroscopic fields being created by them as self-consistent ones [3]. While Emden has derived the equations for the density of stellar matter, Frencel was the one to express them in terms of the potential of gravitational field being formed by the cluster. Trying to solve the problem of the density distribution in a spherical cluster, he had come to an unexpected conclusion that solutions obtained were leading to meaningless physical results.

In this notice it is shown, that the correct choice of the boundary conditions while solving a spherical problem of density distribution in a self-consistent system cannot be an arbitrary one, but is prescribed by the fundamental law of total pressure conservation, existing in the plane-symmetric system.

2. THE MAIN EQUATIONS OF THE PROBLEM

To generalize Emden's paper [1] and to exploit Frencel's approach [3], let us write the three-dimensional equations of gravitational statics in a modern notation of vector analysis

$$\rho\vec{g} + \vec{f} = 0; \tag{2.1}$$

$$div\vec{g} = -4\pi G\rho; \tag{2.2}$$

$$\vec{g} = -grad(\varphi); \tag{2.3}$$

$$p = \rho kT / m; \tag{2.4}$$

$$\vec{f} = -grad(p). \tag{2.5}$$

There ρ is the mass density of an elementary unit, $\vec{g}$ – the strength of the macroscopic gravitational field, p – the pressure inside the system, T – the absolute temperature of the system, φ – the potential of the self-consistent field, G – the gravitation constant, m – the mass of a gravitating particle, k – Boltzmann constant.

The first equation of the system represents the balance condition of an elementary volume of the system of gravitating particle. The second one is the differential form of the Newton's law that describes divergent static fields of smeared mass. The equation (2.3) relates the potential to the strength of the gravitation field, and (2.4) – the equation of state with uniform temperature. The equation (2.5) is the definition of Bernoulli's gas-static force. Notice, that the relation between the strength of the field and the potential (2.3) was not been applied by Emden in his study.

Let us show that the complete system of equations (2.1-2.5) describes the collective interaction among gravitating particles, where the back action of the field on the particles, that generate this field, is manifested.

3. Field Equation of self-consistent Gravitation Theory

Substituting (2.5) and (2.3) into (2.1), we get

$$\rho grad(\varphi) + grad(p) = 0. \tag{3.1}$$

By taking into consideration the equation of state (2.4) and the fact that the temperature is uniform, let us reduce (3.1) to the form

$$grad\left(\frac{m\varphi}{kT} + ln\,\rho\right) = 0. \tag{3.2}$$

It is obvious from (3.2) that any equilibrium of gravitating particles with the field is characterized by the scalar integral

$$\frac{m\varphi}{kT} + ln\,\rho = \frac{m\varphi_0}{kT} + ln\,\rho_0 = const, \tag{3.3}$$

where ρ_0 and φ_0 are constants. Bolzmann's distribution function follows from (3.3)

$$\rho = \rho_0\, exp[-m(\varphi - \varphi_0)/kT]. \tag{3.4}$$

By substituting (3.4) into (2.2) we express everything in terms of φ and convolve the system of equations (2.1) – (2.5) into one equation (i.e., carry out the process of making the system self-consistent)

$$\Delta\varphi = 4\pi G\rho_0\, exp[-m(\varphi - \varphi_0)/kT]. \tag{3.5}$$

The equation (3.5) is a three-dimensional field analog of Emden's E – equation. This equation describes the distribution of macroscopic potential of dynamic systems of particles with uniform temperature. The particles are in a static equilibrium with the self-consistent field. The positive sign of the right-hand side of the equation (3.5) denotes that the system consists of particles that interact according to the Newton's law. The equation (3.5) was first derived by Frencel in [3].

4. FIRST INTEGRAL OF EMDEN'S *E* - EQUATION FOR PLANE-SYMMETRIC CASE

The equation (3.5) for plane-symmetric case has the form

$$\varphi'' = 4\pi G\rho_0\, exp\left[-m(\varphi-\varphi_0)/kT\right], \tag{4.1}$$

where the primes denote derivatives with respect to the x-coordinate.

The order of the equation (4.1) can be reduced. It has a first integral, corresponding to the total pressure of the system [6]:

$$\frac{(\varphi')^2}{8\pi G} + p(\varphi) = P = H(\varphi'/4\pi, \varphi) = const\,. \tag{4.2}$$

where $p(\varphi) = p_0\, exp\left[-m(\varphi-\varphi_0)/kT\right]$ – the pressure of gravitating particles of the system in the plane φ, and $p_0 = \rho_0 kT/m = n_0 kT$ – the pressure of gravitating particles of the system in the plane $\varphi = \varphi_0$.

Total pressure P of the system in (2.4) consists of two terms: the first one represents the pressure of the self-consistent field of the system, and the second one – the gas-kinetic pressure of the particles. The first integral (4.2) coincides with the Hamiltonian function, where canonically conjugated quantities are the generalized momentum $\varphi'/4\pi G$ and φ is the generalized coordinate. Coordinate x acts as the generalized time.

Equation (4.2) is satisfied under the conditions of the absence of any external static gravitational fields, considered with respect to the self-consistent field of the system. The class of even functions and their derivatives is always set in such way that the sum of pressures of the field and particles of the system would be invariant at any plane inside the system.

The conservation law (4.2) also means that in any plane of the system being considered the gradients of self-consistent field and particles of the system are equal and opposite. The volume density of Bernoulli force (further denoted just as Bernoulli force) (2.5) is opposite to the pressure gradient of the particles. It receives a new mathematical definition in the system under consideration. The Bernoulli force 1) has the same value and direction as the pressure gradient of the self-consistent field; 2) balances Newton's forces of gravitation; and 3) provides a class of equilibrium states of particles with the field, being generated by these particles.

Substituting the mass density from the equation (2.2) into (2.1), we derive the connection between the Bernoulli force and the pressure gradient of self-consistent field $\vec{G}_g$

$$\vec{f} = -\rho\vec{g} = \frac{\vec{g}}{4\pi G} div\vec{g} = \vec{G}_g\,, \tag{4.3}$$

which together with (2.5) gives the physical condition of confinement of the substance by the self-consistent field

$$\vec{G}_g + grad(\, p\,)=0. \tag{4.4}$$

Equality (4.4) points to the earlier unknown property of gravitation self-consistent field to hold an inhomogeneous system of particles in a restricted space by static forces of field origin.

It follows from this equality that the system of collectively interacting particles is in a static equilibrium with the self-consistent field of gravitation only when the sum of the gradients of field pressure and particle pressure is equal to zero. This equality has to be satisfied in any arbitrarily elementary volume of the system. The condition (4.4) is rather strict and permits to reject the nonphysical solutions of the system (2.1) – (2.5).

5. DISTRIBUTION OF PHYSICAL PARAMETERS IN A PLANE-SYMMETRIC SYSTEM

After integrating (4.2) using the condition that the potential reaches an extremum $\varphi' = 0$ with the value φ_0 (it's realized, when $P = p_0$) and putting this extremum in the origin of coordinates $x=0$, we derive the law of the distribution of the potential along the coordinate

$$\varphi = \varphi_0 + \frac{2kT}{m} ln\left[ch\left(\frac{x}{l} \right) \right], \tag{5.1}$$

where

$$l = \sqrt{kT/(2\pi G m \rho_0)} \tag{5.2}$$

and it is characteristic spatial scale of the system.

The potential distribution along the coordinate depends on two parameters, as one can see from (5.1). They are the temperature T and potential φ_0. The distribution has a form of a potential well with infinite walls, which has minimum with a value φ_0 in the plane x=0.

The distribution of the projection of the strength of the gravitational self-consistent field along the coordinate of the system is derived according to the law:

$$g_x = -\varphi' = -\frac{2kT}{ml} th(\, x/l\,). \tag{5.3}$$

One can see from (5.3) that the field strength of the system vanishes in the plane $x=0$, and when $x/l \to \pm\infty$ $g_x \to \pm g_*$, where $g_* = 2kT/ml = \sqrt{8\pi G p_0}$ – the scale of the field.

The system is not spatially confined, since the mass density, number density and the pressure of particles have a soliton-like distribution with a maximum at the bottom of the well

$$\frac{\rho}{\rho_0} = \frac{n}{n_0} = \frac{p}{p_0} = ch^{-2}(x/l). \tag{5.4}$$

As it is seen from (5.3) and (5.4), the field pushes out the particles into the regions in which the potential energy of the system is minimal, and the value of the field stays the same in the regions devoid of matter.

The distribution of the field pressure along the coordinate of the system follows from (5.3):

$$D = (\varphi')^2 / 8\pi G = g_*^2 th^2(x/l)/8\pi G. \tag{5.5}$$

One can see from (5.5) and (5.4) that the sum of the pressures of particles and field of the system in any plane of the interaction space stays constant and equal to the total pressure of the system $P=p_0$, which is the integral of the system. By taking a derivative of (5.5) one can show that the pressure gradient of the field in any plane of the system is opposite to the pressure gradient of the particles that is derived from (5.4), and is equal to it in the absolute value:

$$dD/dx = -dp/dx = 2g_*^2 th(x/l)/[8\pi Glch^2(x/l)]. \tag{5.6}$$

The directions of the gradients allow one to ascertain the directions of the volume forces that hold the system examined in balance. Newton's forces, compressing the system of the particles, are directed towards the plane x=0 and coincide with the direction of the $\vec{g}$ vector. Bernoulli forces, expanding the system, are created by the pressure gradient of the self-consistent field (4.3), which balances out the action of the pressure gradient of the particles.

The mathematical equalities (2.5) and (4.3) point the dual role of self-consistent field that generates the configuration of a trap. On one hand, the field creates the pressure gradient in the matter which is aligned with the vector of the field strength (2.3). On the other hand, this field creates the static force (4.3), which balances out the arising gradient.

One can formulate the field boundary conditions that are adequate to the problem under consideration as following: there must be a surface in the system, where the pressure of the self-consistent field vanishes, and where the potential is minimal.

In the plane-symmetric case this point is chosen to be at the origin of the coordinates of the system. In the spherically-symmetric case such a choice would be difficult. As it is shown in the next section, these boundary conditions can be realized at a finite distance from the center of the system.

6. SOLUTIONS OF EMDEN'S E - EQUATION IN A SPHERICALLY SYMMETRIC CASE

Let us write down the equation (3.5) for spherical symmetry, taking into account only the radial dependence of the potential:

$$\varphi'' + 2\varphi'/r = 4\pi G m n_0 \exp[-m(\varphi - \varphi_0)/kT], \tag{6.1}$$

where $n_0 = \rho_0 / m$ is the value of the number density of particles of the system on a sphere $\varphi = \varphi_0$, and primes denote derivatives with respect to r.

Changing the variables in (6.1) to the function $\varphi(r) = -2kTy(x)/m$ of the variable $x=r/R$, where R is the radius of the sphere on which the field boundary conditions are being specified, we reduce (6.1) to the form

$$xy'' + 2y' + \beta^2 x \exp(2y) = 0, \tag{6.2}$$

where

$$\beta^2 = \frac{2\pi G m^2 n_0 R^2}{kT} = \frac{T_*}{T} = \frac{R^2}{l^2} \tag{6.3}$$

is the parameter of state of a spherical system and

$$T_* = \frac{2\pi G m^2 n_0 R^2}{k} \tag{6.4}$$

is the characteristic temperature of the system (primes denote derivatives with respect to x). The parameter of state of a spherical system β can be interpreted in two different ways. On one hand it allows one to compare the temperature of the system with the characteristic one. On the other hand it allows one to compare the radius of the sphere on which the boundary conditions are specified with the spatial scale of the system l (5.2).

Let us look for the solutions of (6.2) with the boundary conditions $x = 1$, $y(1) = 0$, $y'(1) = 0$, which assume the existence of a sphere with zero field pressure in the cluster. Changing the variables to the new function

$$y(x) = \eta(\xi) - \xi, y' = \frac{d\xi}{dx}\left(\frac{d\eta}{d\xi} - 1\right), \text{ where } \xi = \ln x,$$

we derive the equation

$$\frac{d^2\eta}{d\xi^2}+\frac{d\eta}{d\xi}=1-\beta^2 exp(2\eta) \qquad (6.5)$$

with the boundary conditions $\xi=0$, $\eta(0)=0$, $\frac{d\eta}{d\xi}(0)=1$, the order of which can be reduced by the change of variable to the new function

$$p(\eta)=\frac{d\eta}{d\xi};\quad \frac{d^2\eta}{d\xi^2}=p\frac{dp}{d\eta}.$$

The first-order differential equation

$$p\frac{dp}{d\eta}+p=1-\beta^2 exp(2\eta) \qquad (6.6)$$

has the $\eta=0$, $p(0)=1$ boundary conditions. It cannot be integrated in elementary functions.

We have carried out a numerical solution of the equation (6.6) for a set of integral curves that pass through the point at which the boundary condition is set. The solution shows that a singular point of Emden exists when $\eta=\eta_*>0$, in which $p\to 0$ and $dp/d\eta\to -\infty$. Figure 1 shows four integral curves in $p=p(z)$ coordinates, where $z=\eta$. The curve 1 has been calculated for β=0.5; the curve 2 – for β=1.0; the curve 3 – for β=1.5; the curve 4 – for β=2.0. As one can see from figure 1, the position of Emden's singular point depends on the value of β^2. For small β^2 this point is located far from the origin of coordinates $z=\eta=0$. For values β^2>>1 the singular point approaches the origin of coordinates from the right. This allows one to find an approximate solution (6.6) when the condition β^2>>1 is valid.

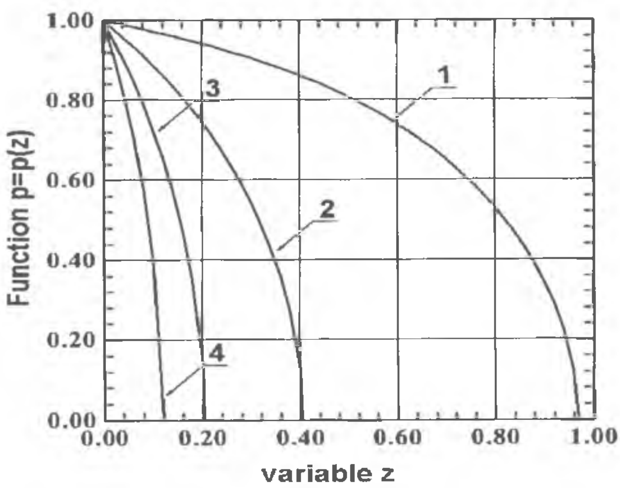

Figure 1. Integral curves p=p (z).

Let us solve (6.6) with respect to derivative:

$$\frac{dp}{d\eta}=\frac{1-\beta^2 exp(2\eta)}{p}-1. \quad (6.7)$$

Under the condition

$$\beta^2 exp(2\eta)/p >> 1/p-1 \quad (6.8)$$

the equation (6.7) can be shortened

$$\frac{dp}{d\eta}=-\frac{\beta^2 exp(2\eta)}{p} \quad (6.9)$$

and integrated

$$p=\sqrt{1-\beta^2[exp(2\eta)-1)]}. \quad (6.10)$$

Let us figure out the β values for which the approximation (6.8) is valid. Let us put (6.8) in the following form:

$$\beta^2 >> (1-p)exp(-2\eta)=f(\eta).$$

The maximal value of the function $f(\eta)$ from the right-hand side part is achieved at the value $\eta=\eta_*$. Then the approximation (6.8) is realized when

$$\frac{1}{\beta^2+1}<<1 \quad (6.11).$$

This condition is satisfied even for β=3 and is improves with β growing.

Coming back to the original function $\varphi(r)$ in (6.10), we derive a two-parameter family of curves, which describe the distribution of the potential along the coordinate of the system in the case when temperature of the system is less than the characteristic one (a cold cluster):

$$\frac{\varphi}{\varphi_*}=\frac{\varphi_0}{\varphi_*}+ln\left[\frac{\beta r}{R\sqrt{\beta^2+1}}ch(A)\right], \quad (6.12)$$

where $\varphi_* = 2kT/m$ is the scale of the potential,

$$A = Arch\left(\frac{\sqrt{\beta^2+1}}{\beta}\right) - ln\left(\frac{r}{R}\right)\sqrt{\beta^2+1},$$

β and φ_0 are the parameters of the distribution.

The expression (6.12) allows one to derive an analytic form of the main gravity-static and kinetic characteristics of a cluster in the case when β^2>>1. The projection of the r-th component of the strength of the self-consistent field is derived from (6.12) and has the following form:

$$\frac{g_r}{g_0} = \frac{RB}{r}, \qquad (6.13)$$

where

$$g_0 = \frac{2kT}{mR}; \quad B = \sqrt{\beta^2+1}th(A) - 1.$$

The distribution of the mass density, the number density and the pressure of the particles in the cluster are the following:

$$\frac{\rho}{\rho_0} = \frac{p}{p_0} = \frac{n}{n_0} = \left[\frac{\beta r}{R\sqrt{\beta^2+1}}ch(A)\right]^{-2}. \qquad (6.14)$$

The pressure of the self-consistent field inside the cluster follows the law:

$$D = \frac{g_r^2}{8\pi G} = \frac{g_0^2 R^2 B^2}{8\pi G r^2}. \qquad (6.15)$$

The projection of the radial component of the gradient of the particle pressure has the form

$$\frac{dp}{dr} = \frac{2p_0 R^2(\beta^2+1)B}{\beta^2 r^3 ch^2(A)}. \qquad (6.16)$$

The derivative of the self-consistent field pressure changes inside the cluster according to the law

$$\frac{dD}{dr} = -\frac{g_0^2 R^2 B}{4\pi G r^3}\left(\frac{\beta^2+1}{ch^2(A)} + B\right). \qquad (6.17)$$

As one can see from (6.12), the radial distribution of the potential depends on two parameters. It has a form of a potential well with infinite walls, which has a minimum $\varphi = \varphi_0$ at the sphere of zero field pressure.

The sphere of the zero field pressure divides the entire interaction space of the cluster in two regions, the interior $0 < r / R < 1$ and the exterior $r / R \geq 1$ one. In the interior region the strength of the self-consistent field and the radius-vector have the same direction. In this region the pressure and the number density of the particles increase while the potential decreases when *r* grows.

In the exterior region the direction of the strength vector of the field is opposite to the direction of the radius-vector. In this region the pressure and the number density of particles decrease, while the potential increases when *r* grows. Inside the cluster the number density of the particles changes smoothly, so the system has no sharp borders.

Since the field pushes out the matter into the region with minimal potential energy, when β^2 is large, a cavity is formed inside the cold cluster, in which the matter is virtually absent. The results obtained in this work refine the solutions that were found in [4]. The monograph can be found at the web site *egf.tsure.ru* (in Russian). The exact solutions of the Emden's E – equation for a cylindrically symmetric case have been derived in [5].

A numerical solution of the exact equation (6.1) with $\varphi(R) = \varphi_0$, $\varphi'(R) = 0$ boundary conditions has been found. Figure 2 shows the spatial distributions of the normalized particle number density n / n_0 of a spherical cluster with uniform temperature for the various parameters of state. The curve 1 has been calculated for the value β=0.5; the curve 2 – for β=0.7; the curve 3 – for β=1.0; the curve 4 – for β=3.0. On this figure one can see the way the filling of the cluster by the particles changes when its temperature is varied in the vicinity of the value $T \sim T_*$. The curve 4 shows that a cavity exists inside the cluster and curves 1, 2, 3 point out the fact that when the temperature grows, the net interaction volume of the cluster is overall being filled by particles and the number of them increases. The number density of particles in the center of gaseous sphere goes to zero for all curves.

7. Estimation of Parameters of Astrophysical Objects

Let us show in the conclusion that the problem solved is related to astrophysical objects. We shall make an estimate for a hollow gaseous sphere, consisting of oxygen, which has a maximum density of ρ_0=1.33 kg/m^3 at the zero field pressure sphere, at a temperature of T=293 K with corresponding pressure 10^5 Pa. Supposes the mass of gravitating particle m is the same as the mass of oxygen molecule 5.32 10^{-26} kg.

For these numbers the spatial scale of the system (5.2) is l=1.2·10^7 m. For a state parameter β=3 the radius of the zero field pressure sphere is R=3.6·10^7 m, which approximately 5 times bigger than the radius of the Earth.

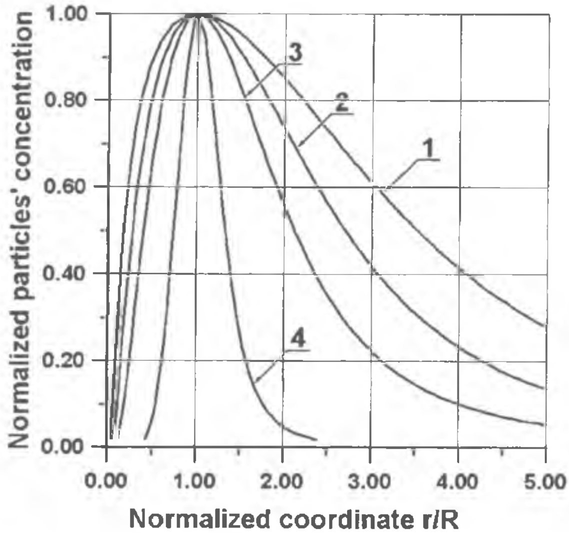

Figure 2. Distribution of particles' concentration in the sphere cluster.

The radius of the cavity is $r_1 = 0{,}6R = 2.16\cdot10^7$ *m* (to make the estimate we cut the number density at the value of n=0.2n_0 – see figure 2). The external radius of the gaseous sphere is $r_2 = 1{,}6R = 5.76\cdot10^7$ *m*. With an average density in the layer $<\rho>$=0.133 *kg/m*^{3} the mass estimate of the hollow gaseous sphere has the value of 10^{23} *kg* that somewhat exceeds the mass of the Moon.

Let us make an estimate for the hollow gaseous sphere consisting of icy nano grains with a diameter of 40 nanometers, with the mass of the grain m=3·10^{-20} *kg* and the maximum density on the zero field pressure sphere ρ_0=80 kg/m^3. The gaseous sphere has a temperature of T=250 *K* with number density $n_0 = 2.7\cdot10^{21}$ m^3.

For these numbers the spatial scale of the system (5.2) is l=1.85·10^3 *m*. For a state parameter β=3 the zero field pressure sphere radius is R=5.6·10^3 *m*. The radius of the cavity is $r_1 = 0{,}6R = 3.3\cdot10^3$ *m*. The external radius of the sphere is $r_2 = 1{,}6R = 8.9\cdot10^3$ *m*. With an average density of the layer $<\rho>$=8 *kg/m*^{3} the estimate of the mass of the sphere has a value of 22 billion ton.

Such a cosmic “snowball” with a size of the order of 20 *km* can not explode during the fall. Descending with the velocity of the order of 40 *km/s* towards the surface of the Earth, it can do vast destructions very similar to those that have been done by the Tunguska phenomenon. Since the sphere has an internal cavity devoid of nano grains, the flux density of the particles incident upon the surface will be substantially lower in the center of the impact than in the neighbouring layers [4]. This will cause minimal destructions at the epicenter of the impact.

CONCLUSION

The conception of collective interaction of the gravitating particles in the system with the uniform temperature has been founded. The complete system of equations describes such interaction among the gravitating particles where the back action of the field on the particles that generate this field is manifested.

The back action of the field on the particles shows the existence of Bernoulli force, which has the field origin, in the system. This force has the same direction as the pressure gradient of the self-consistent field and compensates the Newton's forces of gravitation in any arbitrary elementary volume of the system. Otherwise, the system of collective interaction of the particles is in a static equilibrium with the self-consistent field of gravitation only when the sum of pressure gradient of the field and pressure gradient of the particles is equal to zero in any arbitrary elementary volume of the system.

Therefore, the first integral of a plane-symmetric system consists of two terms. The first one represents the pressure of the self-consistent field of the system, and the second one is the gas-kinetic pressure of the particles. The distributions of potential along the coordinate depend on two parameters, which are the temperature and the minimal value of the potential. They represent the potential wells with infinite walls, which have the minimal value of the potential at the bottom of the well. The field pushes out the particles into the region with the minimal potential energy of the system and remains the uniform one in the regions devoid of matter. The exact solution, obtained in the plane-symmetric case, allows one to specify the boarder conditions, which are adequate to the problem under consideration, i.e. there must be a surface in the system where the pressure of the field vanishes, and the potential is minimal.

The approximate solution in a spherically symmetric case for the border conditions of the field is derived only when the temperature of the system is less than the characteristic one (a cold cluster). This solution demonstrates that the radial distribution of potential also depends on two parameters. It has a form of potential well with infinite walls, which has a minimum on the sphere of zero field pressure, and the radius of the sphere is finite. In this case the field pushes out the matter into the region with minimal potential energy too. For the large values of the state parameter the cavity is formed inside the cold cluster, in which the matter is virtually absent. The numerical solution of exact equation points on the fact that the number density of the particles in the center of the system goes to zero for any values of the state parameter.

The self-consistent field, which generates the configuration of a trap, plays dual role. On one hand, the field creates the pressure gradient in the matter, which is aligned with the vector of the field strength. On the other hand, this field creates the static Bernoulli force, which balances out the arising gradient.

The conception of the collective interaction formulated is related to astrophysical objects. The estimations executed for the hollow gaseous balls give the proportions observed of the systems. The hypothesis that the Tunguska phenomenon is a hollow cosmic "snowball" with the huge mass, consisting of icy nano grains, has been expressed. The destructions have been done in the epicenter by the impact of such "snowball" are minimal due to the cavity inside this "snowball".

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