Physical and Mathematical Consistency of the Janus Cosmological Model (JCM)

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The Janus Cosmological Model is based on a system of two coupled field equations. It explains the nature of dark matter and dark energy with negative mass and without the runaway paradox that arises in general relativity. We first recall how this system was built, from a simple Newtonian toy model to a relativistic bimetric theory, that is now improved in order to fulfill mathematical constraints and set up on a Lagrangian derivation.

1 The long genesis of the Janus Cosmological Model

Roots of the Janus Cosmological Model are like assembling different pieces of a puzzle. There are indeed several starting points for this bimetric approach. The first is the missing primordial antimatter, a problem solved in 1967 by Andrei Sakharov in [1] with the representation of the universe not as a single entity born from the beginning of time, but two spacetimes with opposite arrows of time communicating only through their common initial singularity, forming a "twin universe" in complete CPT symmetry, as represented in the didactic Figure 1.

Then, the first step is to consider that these two entities can interact gravitationally, which is equivalent to folding the object of Figure 1 on itself as in Figure 2.

In 1977, a first modeling using non relativistic theoretical tools is attempted in [2] and [3] with two Boltzmann equations coupled with Poisson's equation. We then realize Sakharov's seminal idea of a complete CPT symmetry between these two entities, an idea also independently used by other authors recently [4]. Such work suggests that a profound paradigm shift involving geometrical grounds should be performed.

Early 1990's, we explore, through computer simulations, what could emerge from interaction laws associated with a mix of positive and negative point masses, according to the following assumption. Interactions laws:

- Like masses attract, according to Newton's law.

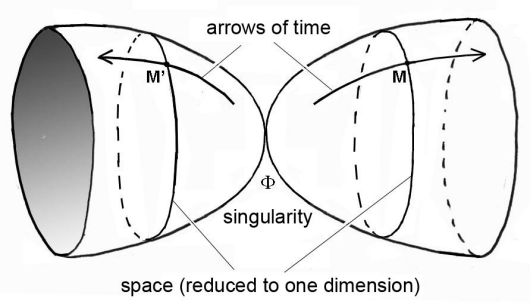


Fig. 1: 2D representation of Sakharov's twin universe model.

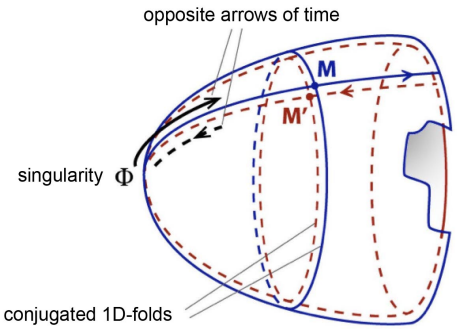


Fig. 2: Sakharov's model with "conjugate folds".

- Unlike masses repel, according to "anti-Newton".

At this stage, it is only a toy model. In 1992, first 2D simulations of two populations with opposite mass and same absolute value of density show a separation of the two entities, as shown in [5], a result reproduced below in Figure 3.

The purpose was to account for the large-scale structure of the universe, which admittedly wasn't a tight fit with these early experiments. But if we now introduce asymmetry in the two mass densities, taking a greater density for the negative mass species, then this population has a shorter Jeans time, hence it is the first to coalesce into conglomerates, by gravitational instability.

$$\text{if } |\rho^{(-)}| \gg \rho^{(+)} \quad \Rightarrow \quad t_{j^{(-)}} = \frac{1}{\sqrt{4\pi G\,|\rho^{(-)}|}} \ll t_{j^{(+)}} = \frac{1}{\sqrt{4\pi G\,\rho^{(+)}}} \tag{1}$$

Following simulations confirm this second hypothesis as they produce an evolution of the positive mass distribution into a large-scale structure with big negative mass conglomerates (optically invisible) repelling the positive mass matter in the remnant space around them as shown in [6], a decisive result reproduced below in Figure 4, this time in very good agreement with the observation of the lacunar, foam-

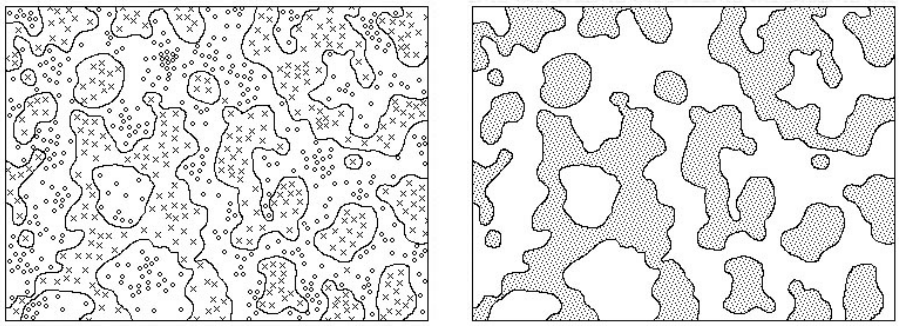


Fig. 3: Flocculation and percolation phenomena between two populations of opposite mass and same overall density. Right: Showing the optically-visible positive mass matter only.

like structure of the universe, where galaxies, clusters and superclusters are organized as a web of filaments, walls and nodes distributed around giant repulsive cosmic voids.

Same approach but different boundary conditions in [7], reproduced in Figure 5.

Such a scenario also produces, in 3D, a mechanism helping galaxy formation along. Indeed, after recombination, if large volumes of gas can coalesce into giant conglomerates, then a problem arises: how to dissipate such enormous gravitational energy transformed into heat? Considering an object of radius R, the amount of energy collected varies according to R^3 while the surface of the heatsink varies as R^2. Therefore, larger masses have a more important cooling time. But the constitution of the large-scale structure suggested by these simulations leads to a compression of the positive mass which distributes according to walls (as observed) that are actually sandwiched between two repulsive conglomerates of negative mass. A strong compression of the positive mass occurs in such planar structures, which are optimal for a quick radiative dissipation of energy, as explained in [6].

Besides 2D simulations, an effective confinement of galaxies despite their high peripheral velocity is analytically demonstrated using an exact solution of two Vlasov equations coupled with Poisson's equation, using the methodology exposed in [5]. The flat rotation curve obtained from such a solution, made possible by the repulsive effect of the surrounding negative mass, has been shown for the first time in [6], a curve reproduced in Figure 6. It is worth noting that such a typical rotation curve has been similarly obtained more recently using the same repulsive action of a negative mass distribution around galaxies, but from 3D computer simulations made by an independent researcher [8].

Using the exact solution of the analytical set of two Vlasov equations coupled with Poisson's equation (image of a 2D galaxy confined by a repulsive negative mass environment), we show in numerical simulations that the rotational motion of the galaxy generates a good-looking barred spiral structure in a few turns (1992 DESY results, published in [6] and [7]).

In order to progress beyond a simple toy model that opens up interesting prospects thanks to the various above-mentioned positive results, it was still necessary at that time to derive interaction laws from a coherent mathematical formalism. The introduction of negative mass in cosmology had been considered as soon as the 1950s, using general relativity, defined by the well-known Einstein field equations which may be written, with a zero cosmological constant:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = +\chi T_{\mu\nu}. \qquad (2)$$

Let's notice that Einstein's equation describes the motion of point masses embedded in a given mass-energy field $T_{\mu\nu}$ along geodesics that derive from a single metric $g_{\mu\nu}$. Then, one gets Bondi's result from [9]. Interaction laws with a single metric:

- Positive masses attract everything.
- Negative masses repel everything.

Which inevitably produce the preposterous "runaway motion" paradox (see Figure 8), a term coined by Bonnor in [10].

Nonetheless, a few authors (Farnes [8], Chardin [11]) still consider that it is possible to introduce negative mass in cosmology keeping the general relativity framework, hence putting up with such phenomenon; despite the fact that the runaway motion has been associated with the possibility of perpetual motion machines since the 1950s, as discussed by Gold with Bondi, Bergmann and Pirani in [12].

On the contrary, from 1995 in [13] we propose a bimetric description of the universe with two coupled metrics and which produce trajectories along their own geodesics, for positive and negative mass particles, respectively. Then, the classical Schwarzschild solution allows, by simply reversing the integration constant, to get trajectories suggesting a gravitational repulsion of positive masses by a negative mass, and vice versa:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \dfrac{2GM}{rc^2}} - r^2 d\theta^2 - \sin^2\theta d\varphi^2, \quad (3)$$

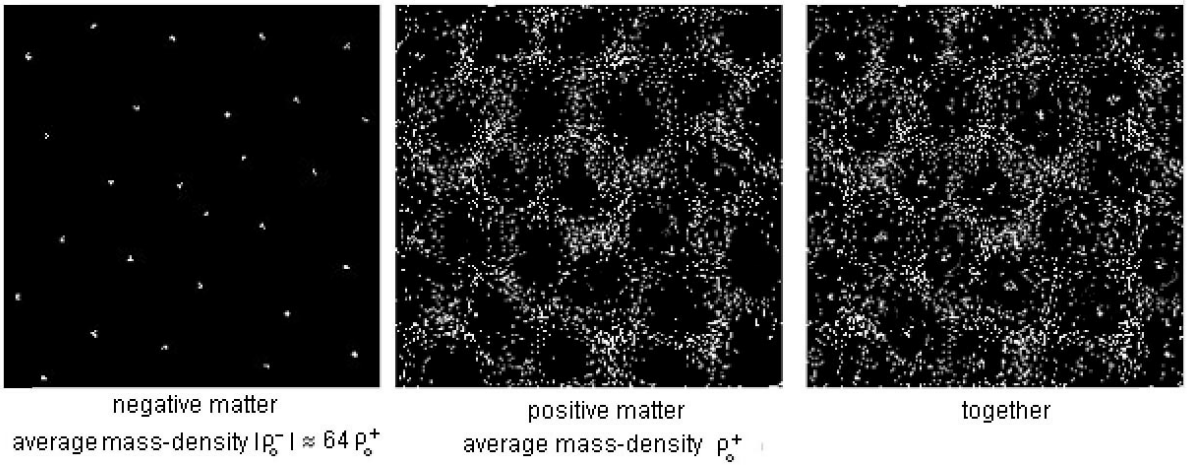


Fig. 4: Result of a 2D large-scale structure simulation [6].

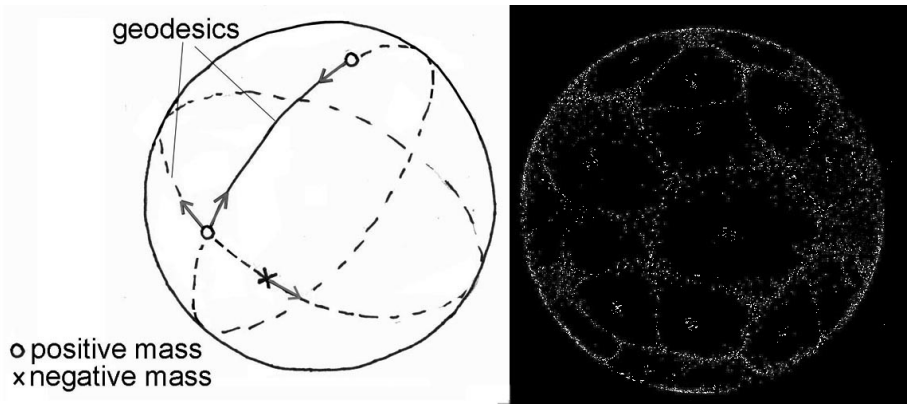


Fig. 5: Result of a 2D large-scale structure simulation on a 2-sphere [7].

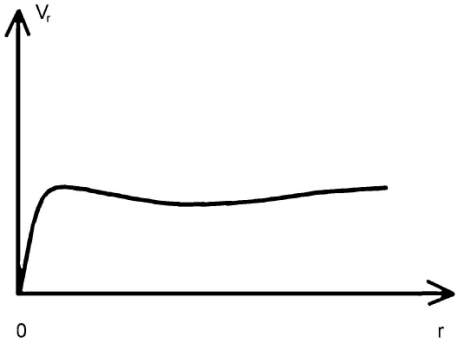


Fig. 6: Flat rotation curve of a galaxy surrounded by a negative mass distribution [6].

$$ds^2 = \left(1+\frac{2GM}{rc^2}\right)c^2dt^2 - \frac{dr^2}{1+\frac{2GM}{rc^2}} - r^2d\theta^2 - \sin^2\theta d\varphi^2. \quad (4)$$

Exploiting this idea, we introduce the concept of negative (diverging) gravitational lensing in the same paper [13]. Considering that a gap within a negative mass distribution is equivalent to a positive mass concentration, we suggest to attribute the strong gravitational lensing effects, observed in the vicinity of galaxies and galaxy clusters, not to a dark matter halo made of positive mass, but instead to their negative mass environment.

From 1994, we also suggest in [5] that such a bimetric description could result from the combination of two Lagrangian densities, due to two Ricci scalars $R^{(+)}$ and $R^{(-)}$. In 2001 [6], we proposed for the first time a system of two coupled field equations, which can be written as:

$$R^{(+)}_{\mu\nu} - \frac{1}{2}R^{(+)}g^{(+)}_{\mu\nu} = +\chi\left[T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu}\right], \quad (5)$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2}R^{(-)}g^{(-)}_{\mu\nu} = -\chi\left[T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu}\right], \quad (6)$$

whose purpose was to account for the postulated interaction laws. Indeed, we make such laws emerge from a dual Newto-

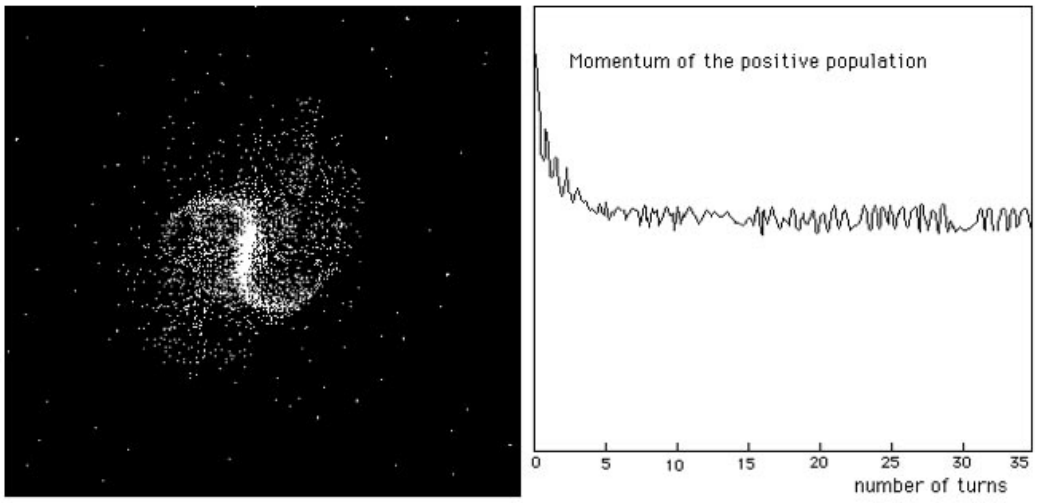


Fig. 7: 2D barred spiral structure [6, 7].

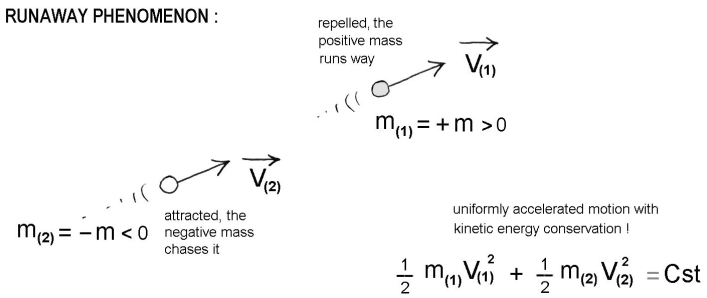


Fig. 8: Runaway motion in general relativity.

nian approximation of this system of two coupled equations. Depending locally on the type of dominant species in a given region of space, equations with no RHS produce solutions of type 36 or 37.

Aforementioned results of simulations showed that an asymmetry in the mass densities of the positive vs negative mass species is required to account for observations of the large-scale structure of the universe. Such density asymmetry can be caused, not because of a larger quantity of negative mass, but if the two space gauge factors $a^{(+)}$ and $a^{(-)}$ are different. Alas, at this level it is impossible to produce a time-dependent solution with $a^{(+)} \neq a^{(-)}$. Inconsistency becomes inevitable when FRLW metrics are introduced in the two field equations: similarly to Friedmann solutions, they produce a couple of differential equations in $a^{(+)}$, $a'^{(+)}$, $a''^{(+)}$ on one hand, and in $a^{(-)}$, $a'^{(-)}$, $a''^{(-)}$ on the other. In the calculation based on Einstein's equations, compatibility betweeen two equations leads to the relation $\rho a^3 = cst$ in the matter-dominated era, which expresses mass-energy conservation. In the bimetric framework of the Janus model based on the two coupled equations 5 and 6, such compatibility reduces the time-dependent solution to $a^{(+)} = a^{(-)}$.

Still in the same 2001 paper [6], we establish the connection between Sakharov's seminal work about two universes with opposite arrows of time, and negative gravity, using dynamical group theory from [14], which shows that time reversal goes with energy inversion, hence mass inversion as $-m = -E/c^2$. We then introduce the "Janus group" to handle the electric charge in a five-dimensional spacetime:

$$\begin{pmatrix} \lambda\mu & 0 & 0 \\ 0 & \lambda\,, L_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{with } \lambda = \pm 1 \quad \text{and } \mu = \pm 1, \quad (7)$$

where L_0 is the component of the orthochronous (forward in time) subset of the Lorentz group. It is the extension of the Poincaré group to five dimensions, which describes the existence of two different kinds of antimatter: one being C-symmetric with respect to normal matter, it has a positive mass; while the other antichronous (backward in time) antimatter is PT-symmetric and has a negative mass. Therefore,

the CPT theorem has to be reconsidered, since the exclusion of negative energy states follows on from an a priori axiom in quantum field theory, which postulates that the operator T has to be antiunitary and antilinear, a hypothesis not necessarily true as shown in [15].

Sakharov's conditions in [1] states that the baryon creation rate from an excess of quarks has been faster than the antibaryon creation rate from fewer antiquarks at $t > 0$, but such CP violation is opposite for $t < 0$ (the "initial singularity" triggering complete CPT reflections) thereby preserving the global symmetry of the whole universe. This allows to define the true nature of the invisible antichronous components of the universe: these are copies of antiparticles that are usually made in a lab, but with negative energy and mass, due to T-symmetry.

The invisibility of such objects is deduced from the idea that PT-symmetric antiparticles emit negative energy photons that follow null-geodesics of their own metric $g_{\mu\nu}^{(-)}$ hence escape detection by optical instruments that are made of positive mass matter.

In 2002, Damour and Kogan in [16] situate the issue with massive bigravity theories, where bimetry covers a different approach. In such models, two branes interact using various massive gravitons (hence the name) with a mass spectrum. The authors propose a Lagrangian derivation, based on an action, which leads to a system of two coupled field equations. But such a model, although mathematically consistent, does not stand up to scrutiny as it does not provide any solution able to be confronted with observations. As it has not been further pursued, it cannot answer this question.

On the other hand, in 2008 and 2009, Hossenfelder in [17] and [18] builds her own bimetric model involving negative mass, from a Lagrangian derivation where she produces a system of two coupled field equations. This time, LHS are identical to the system (5;6) which follows on from the presence of terms $R^{(+)}\sqrt{g^{(+)}}$ and $R^{(-)}\sqrt{g^{(-)}}$ in the Lagrangian densities considered. Exploiting her Lagrangian derivation, she reveals the determinant ratios of the two metrics $\sqrt{g^{(+)}/g^{(-)}}$ and $\sqrt{g^{(-)}/g^{(+)}}$ that had already been pointed out in previous work [19] and [20]. She finally tackles two Friedmann solutions, without confronting them to observational data. Actually, although sharing many similarities, having the same kind of coupled field equations regarding negative mass, a fundamental difference remains between Hossenfelder's bimetric theory and the Janus Cosmological Model.

Indeed, Hossenfelder doubts that the second entity can have an important effect on the distribution of standard matter, qualifying the gravitational coupling between the two species as "extremely weak". This is because "for symmetry reason" she considers that the absolute values of the mass density of the two populations should be of the same order of magnitude. Such hypothesis leads to a global zero field configuration, which does not fit with observations, as she notices. Then, examination of possible fluctuations seems to be her main concern. Not perceiving that a *profound dissymmetry* is on the contrary the key to the interpretation of many phenomena, including the acceleration of the cosmic expansion, she will not develop her model further during the following decade, focusing instead on other research topics.

Nonetheless, Hossenfelder points out a "smoking gun signal" that could highlight the existence of invisible negative mass in the universe, through the detection of diffracted light rays caused by diverging lensing, an effect previously predicted in [13]. We indeed showed from 1995 that photons emitted by high redshift galaxies ($z > 7$) are diffracted by the presence of invisible conglomerates of negative mass on their path. This reduces the apparent magnitude of such galaxies, making them appear as *dwarf*, which is consistent with observations.

In 2014 in [21] we take again the system (5;6) and attempt to modify it according to:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = +\chi \left[T_{\mu\nu}^{(+)} + \varphi T_{\mu\nu}^{(-)} \right], \tag{8}$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi \left[\phi T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)} \right]. \tag{9}$$

Introducing two functions $\varphi()$ and $\phi()$ that allow a time-dependent homogeneous and isotropic solution, so that $a^{(+)} \neq a^{(-)}$. This is possible by switching to the system:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = +\chi \left[T_{\mu\nu}^{(+)} + \left(\frac{a^{(-)}}{a^{(+)}} \right)^3 T_{\mu\nu}^{(-)} \right], \tag{10}$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi \left[\left(\frac{a^{(+)}}{a^{(-)}} \right)^3 T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)} \right]. \tag{11}$$

We obtained such a result by assuring energy conservation, not by deriving these equations from the system proposed in [18]. From (10;11) we then build an exact solution involving a large asymmetry, so that $|\rho^{(-)}| \gg \rho^{(+)}$, accounting fot the acceleration of the expansion of the universe. D'Agostini thereafter showed in 2018 in [22] that this exact solution is in very good agreement with latest observational data. In parallel we published in 2014 in [23] a Lagrangian derivation based on the functional relation:

$$\delta g^{(-)\mu\nu} = -\delta g^{(+)\mu\nu}, \tag{12}$$

giving the following system of two coupled field equations:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = +\chi \left(T_{\mu\nu}^{(+)} + \sqrt{\frac{-\mathbf{g}^{(-)}}{-\mathbf{g}^{(+)}}}\, T_{\mu\nu}^{(-)} \right), \tag{13}$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi \left(T_{\mu\nu}^{(-)} + \sqrt{\frac{\mathbf{g}^{(+)}}{\mathbf{g}^{(-)}}}\, T_{\mu\nu}^{(+)} \right), \tag{14}$$

which is similar to Hossenfelder's system in her previous Lagrangian derivation [18], although both constructions are completely different. In our derivation, the square root in the determinant ratio of the metrics directly follows on from hypothesis (14). Let's recall that such a ratio always appears as soon as a bimetric approach is attempted, see for example [19] and [20]. Admittedly however, we cannot rule out that the system (15);(16), as well as the newer one exposed hereinbelow, can be considered as a particular case of Hossenfelder's own model.

In 2014 in [23] we extend the Janus framework to a class of solutions where the two speeds of light and, in the positive and negative sectors, are different. In 2018 in [25] we propose to evaluate the magnitude of their ratio, based on a study of the fluctuations in the CMB, which leads to the following conclusion:

$$\frac{a^{(-)}}{a^{(+)}} \simeq \frac{1}{100}, \qquad \frac{c^{(-)}}{c^{(+)}} \simeq \frac{1}{10}. \tag{15}$$

The combination of such different space scale factors and speeds of light would allow a gain factor of 1000 in travel time, regarding a hypothetical technology making apparent FTL interstellar travel by mass inversion possible, as evoked in [23] and [26].

The paper [23] then summarizes many observational data in good agreement with features of the Janus Cosmological Model.

2 The 2014 JCM and the Bianchi identities

From 2014, the Janus system of two coupled field equations (13; 14) satisfies the Bianchi identities, either trivially when the RHS are zero, or when one considers time-dependent homogeneous and isotropic solutions. However, inconsistency appears when one tries to describe with this system a time-independent situation with a spherical symmetry, modeling a star of constant density surrounded by a vacuum. Thus, a new modification of the equation system must be considered, as explained below.

Let's consider a portion of the universe where one of the two species is absent, e.g. the negative energy species, repelled away by a local concentration of positive mass. Let's limit our analysis to the search of a time-independent solution for a spherically symmetric system, and Newtonian approximation. The corresponding system is:

$$R^{(+)}_{\mu\nu} - \frac{1}{2} R^{(+)} g^{(+)}_{\mu\nu} = +\chi T^{(+)}_{\mu\nu}, \tag{16}$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2} R^{(-)} g^{(-)}_{\mu\nu} = -\chi T^{(+)}_{\mu\nu}. \tag{17}$$

Then the two metrics have the form:

$$ds^{(+)2} = e^{\nu^{(+)}} c^2 dt^2 - e^{\lambda^{(+)}} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2, \tag{18}$$

$$ds^{(-)2} = e^{\nu^{(-)}} c^2 dt^2 - e^{\lambda^{(-)}} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2. \tag{19}$$

We consider a sphere whose radius r_s is filled by matter of constant density $\rho^{(+)}$ surrounded by vacuum. Outside of the sphere, the two metrics are:

$$ds^{(+)2} = \left(1-\frac{2m}{r}\right) c^2 dt^2 - \frac{dr^2}{1-\dfrac{2m}{r}} - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2, \tag{20}$$

$$ds^{(-)2} = \left(1+\frac{2m}{r}\right) c^2 dt^2 - \frac{dr^2}{1+\dfrac{2m}{r}} - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2, \tag{21}$$

with:

$$m = \frac{G}{c^2}\frac{4\pi r_s^3}{3}\rho^{(+)}. \tag{22}$$

We can write the stress-energy tensor as:

$$T^{(+)\nu}_{\ \ \mu} = \begin{pmatrix} \rho^{(+)} & 0 & 0 & 0 \\ 0 & -\frac{p^{(+)}}{c^2} & 0 & 0 \\ 0 & 0 & -\frac{p^{(+)}}{c^2} & 0 \\ 0 & 0 & 0 & -\frac{p^{(+)}}{c^2} \end{pmatrix}, \tag{23}$$

where $p^{(+)}$ is the pressure insides the star of radius r_s filled with constant density $\rho^{(+)}$. Equations (16) and (17) give the following differential equations:

$$p^{(+)\prime} = -\left(\rho^{(+)}c^2 + p^{(+)}\right)\frac{m(r) + 4\pi G p^{(+)} r^3/c^4}{r\,(r-2m(r))}, \tag{24}$$

$$p^{(+)\prime} = +\left(\rho^{(+)}c^2 + p^{(+)}\right)\frac{m(r) + 4\pi G p^{(+)} r^3/c^4}{r\,(r+2m(r))}, \tag{25}$$

where:

$$m(r) = \frac{G}{c^2}\frac{4\pi r^3}{3}\rho^{(+)}. \tag{26}$$

After Newtonian approximation:

$$p^{(+)} \ll \rho^{(+)}c^2 \quad , \quad r \gg 2m, \tag{27}$$

which gives:

$$p^{(+)\prime} = -\frac{\rho^{(+)}c^2 m(r)}{r^2}, \tag{28}$$

$$p^{(+)\prime} = +\frac{\rho^{(+)}c^2 m(r)}{r^2}. \tag{29}$$

So that we get a physical and mathematical contradiction, that must be cured.

3 Lagrangian derivation of a new JCM, as of 2019

Consider the two diagonal constant matrices:

$$I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \varphi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{30}$$

$$S = \int_{D4}\left[IR^{(+)}\sqrt{-g^{(+)}} + \varphi R^{(-)}\sqrt{-g^{(-)}} - \chi\,(I+\varphi)\,L^{(+)}\sqrt{-g^{(+)}} + \chi\,(I+\varphi)\,L^{(-)}\sqrt{-g^{(-)}}\right]\mathrm{d}^4x \tag{31}$$

$$\delta\int_{D4} R^{(+)}\sqrt{-g^{(+)}}\;\mathrm{d}^4x = \int_{D4}\left(R^{(+)}_{\mu\nu} - \frac{1}{2}R^{(+)}g^{(+)}_{\mu\nu}\right)\sqrt{-g^{(+)}}\;\delta g^{(+)\mu\nu}\;\mathrm{d}^4x \tag{32}$$

$$\delta\int_{D4} R^{(-)}\sqrt{-g^{(-)}}\;\mathrm{d}^4x = \int_{D4}\left(R^{(-)}_{\mu\nu} - \frac{1}{2}R^{(-)}g^{(-)}_{\mu\nu}\right)\sqrt{-g^{(-)}}\;\delta g^{(-)\mu\nu}\;\mathrm{d}^4x \tag{33}$$

$$\delta\int_{D4} L^{(+)}\sqrt{-g^{(+)}}\;\mathrm{d}^4x = \int_{D4} T^{(+)}_{\mu\nu}\sqrt{-g^{(+)}}\;\delta g^{(+)\mu\nu}\;\mathrm{d}^4x \tag{34}$$

$$\delta\int_{D4} L^{(-)}\sqrt{-g^{(-)}}\;\mathrm{d}^4x = \int_{D4} T^{(-)}_{\mu\nu}\sqrt{-g^{(-)}}\;\delta g^{(-)\mu\nu}\;\mathrm{d}^4x \tag{35}$$

$$ds^{(+)2} = \left(1 - \frac{8\pi G\,r_s^3\,\rho^{(+)}}{c^2}r\right)c^2dt^2 - \left(1 + \frac{8\pi G\,r_s^3\,\rho^{(+)}}{c^2}r\right)dr^2 - r^2d\theta^2 - \sin^2\theta d\varphi^2 \tag{36}$$

$$ds^{(-)2} = \left(1 + \frac{8\pi G\,r_s^3\,\rho^{(+)}}{c^2}r\right)c^2dt^2 - \left(1 - \frac{8\pi G\,r_s^3\,\rho^{(+)}}{c^2}r\right)dr^2 - r^2d\theta^2 - \sin^2\theta d\varphi^2 \tag{37}$$

$$\delta g^{(+)}_{00} = -\frac{8\pi G\,r_s^3\,\rho^{(+)}}{c^2}\,r\,\delta\rho^{(+)} = -\delta g^{(-)}_{00} \qquad \delta g^{(+)}_{11} = -\frac{8\pi G\,r_s^3\,\rho^{(+)}}{c^2}\,r\,\delta\rho^{(+)} = -\delta g^{(-)}_{11} \tag{38}$$

Introducing the action (eq. 31) and performing the following bivariation, taking account of $I\varphi = \varphi$ and $\varphi\varphi = I$, results in equations 32–35.

From a previous Lagrangian derivation [7] :

$$\delta g^{(-)\mu\nu} = -\delta g^{(+)\mu\nu} \tag{39}$$

Our goal: to set up a system of two coupled field equations providing joint solutions corresponding to Newtonian approximation. In such conditions the external metrics are given in equations (36) and (37).

We may consider that such metrics belong to subsets of Riemannian metrics with signature $(+---)$ which obey relationship (39) (see eqs. (38)). If we consider that (39) defines joint metrics, they obey:

$$R^{(+)}_{\mu\nu} - \frac{1}{2}R^{(+)}\,g^{(+)}_{\mu\nu} = +\chi\left(T^{(+)}_{\mu\nu} + \sqrt{\frac{-\mathbf{g}^{(-)}}{-\mathbf{g}^{(+)}}}\,\varphi\,T^{(-)}_{\mu\nu}\right), \tag{40}$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2}R^{(-)}\,g^{(-)}_{\mu\nu} = -\chi\left(T^{(-)}_{\mu\nu} + \sqrt{\frac{\mathbf{g}^{(+)}}{\mathbf{g}^{(-)}}}\,\varphi\,T^{(+)}_{\mu\nu}\right). \tag{41}$$

4 Back to the star model

Starting from the new joint system (40);(41) we obtain the analogous of the system (16);(17) where, in the second equation, we would replace the tensor $T^{(+)}\,g^{(+)}_{\mu\nu}$ by $\hat{T}^{(+)}_{\mu\nu}$, so that:

$$\hat{T}^{(+)}_{00} = T^{(+)}_{00} = \rho^{(+)}, \tag{42}$$

$$\hat{T}^{(+)}_{ii} = -T^{(+)}_{ii} \qquad \text{with} \qquad \mathrm{j} = \{1,2,3\}, \tag{43}$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2}\,R^{(-)}\,g^{(-)}_{\mu\nu} = -\chi\hat{T}^{(+)}_{\mu\nu}. \tag{44}$$

With the joint metrics (18) and (19), inside the star, plus compatibility conditions satisfying (20) and (21) at its border $r = r_s$ we get the following result:

$$p^{(+)\prime} = -\left(\rho^{(+)}c^2 + p^{(+)}\right)\frac{m(r) + 4\pi G p^{(+)}r^3/c^4}{r\,(r - 2m(r))}, \tag{45}$$

$$p^{(+)\prime} = -\left(\rho^{(+)}c^2 - p^{(+)}\right)\frac{m(r) - 4\pi G p^{(+)}r^3/c^4}{r\,(r + 2m(r))}, \tag{46}$$

with $m(r)$ given by (26).

Equation (45) is nothing but the famous Tolman-Oppenheimer-Volkoff equation.

Applying the Newtonian approximation, any inconsistency vanishes. Such equations mean that inside the star, the pressure counterbalances the gravitational pull. The geodesics are given by equations (48) and (49), with:

$$\hat{R}^2 = \frac{3c^2}{8\pi G\rho^{(+)}}. \tag{47}$$

Linearizing leads to equations (50) and (51). Notice that equation (52) fits (39).

$$ds^{(+)2} = \left[\frac{3}{2}\sqrt{1-\frac{r_s^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1-\frac{r^2}{\hat{R}^2}}\right]^2 c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1-\dfrac{r^2}{\hat{R}^2}} - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2 \tag{48}$$

$$ds^{(-)2} = \left[\frac{3}{2}\sqrt{1+\frac{r_s^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1+\frac{r^2}{\hat{R}^2}}\right]^2 c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1+\dfrac{r^2}{\hat{R}^2}} - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2 \tag{49}$$

$$ds^{(+)2} = \left(1 - \frac{3}{2}\frac{r_s^2}{\hat{R}^2} + \frac{1}{2}\frac{r^2}{\hat{R}^2}\right)c^2\mathrm{d}t^2 - \left(1 + \frac{3}{2}\frac{r_s^2}{\hat{R}^2} - \frac{1}{2}\frac{r^2}{\hat{R}^2}\right)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{50}$$

$$ds^{(-)2} = \left(1 + \frac{3}{2}\frac{r_s^2}{\hat{R}^2} - \frac{1}{2}\frac{r^2}{\hat{R}^2}\right)c^2\mathrm{d}t^2 - \left(1 - \frac{3}{2}\frac{r_s^2}{\hat{R}^2} + \frac{1}{2}\frac{r^2}{\hat{R}^2}\right)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{51}$$

$$\delta g_{00}^{(+)} = -\frac{4\pi G\left(3r_s^3 - r^2\right)}{3c^2}\,\delta\rho^{(+)} = -\delta g_{00}^{(-)} \qquad \delta g_{11}^{(+)} = -\frac{4\pi G\left(3r_s^3 - r^2\right)}{3c^2}\,r\,\delta\rho^{(+)} = -\delta g_{11}^{(-)} \tag{52}$$

5 Back to our basic assumption: $\delta g^{(-)\mu\nu} = -\delta g^{(+)\mu\nu}$

The time-dependent joint solutions presented in [21] correspond to the following FRLW metrics:

$$\mathrm{d}s^{(+)2} = \left(dx^0\right)^2 - a^{(+)^2}\frac{\mathrm{d}u^2 + u^2\,\mathrm{d}\theta^2 + u^2\,\sin^2\theta\,\mathrm{d}\varphi^2}{\left(1 + \dfrac{k^{(+)}\,u^2}{4}\right)^2}, \tag{53}$$

$$\mathrm{d}s^{(-)2} = \left(dx^0\right)^2 - a^{(-)^2}\frac{\mathrm{d}u^2 + u^2\,\mathrm{d}\theta^2 + u^2\,\sin^2\theta\,\mathrm{d}\varphi^2}{\left(1 + \dfrac{k^{(-)}\,u^2}{4}\right)^2}, \tag{54}$$

which give, with the single solution $k^{(+)} = k^{(-)} = -1$:

$$a^{(+)2}\frac{\mathrm{d}^2a^{(+)}}{(\mathrm{d}x^0)^2}, -\frac{8\pi G\rho_0}{3c_0^2} = 0 \tag{55}$$

$$a^{(-)2}\frac{\mathrm{d}^2a^{(-)}}{(\mathrm{d}x^0)^2}. + \frac{8\pi G\rho_0}{3c_0^2} = 0 \tag{56}$$

Whose exact parametric solutions are, for (55):

$$x^0 = \frac{4\pi G\rho_0}{3c_0^2}\left(1 + \frac{sh(2v)}{2} + v\right), \tag{57}$$

$$a^{(+)} = \frac{4\pi G\rho_0}{3c_0^2}ch^2(v), \tag{58}$$

and for (56):

$$x^0 = \frac{4\pi G\rho_0}{3c_0^2}\left(sh(2w) - 2\,w\right), \tag{59}$$

$$a^{(-)} = \frac{4\pi G\rho_0}{3c_0^2}\left(ch^2(w) - 1\right). \tag{60}$$

Let's compute the variations $\delta g_{\mu\nu}^{(+)}$ and $\delta g_{\mu\nu}^{(-)}$ under a variation $\delta\rho_0$ of their single parameter, the dominant matter density ρ_0. The variations $\delta g_{00}^{(+)}$, $\delta g_{00}^{(-)}$, $\delta g_{11}^{(+)}$, $\delta g_{11}^{(-)}$ depend on the factors $a^{(+)}\delta a^{(+)}$ and $a^{(-)}\delta a^{(-)}$. But we have:

$$\begin{aligned}&\frac{\mathrm{d}a^{(+)}}{\mathrm{d}x^0} = th(v),\\ &\frac{\mathrm{d}^2a^{(+)}}{(\mathrm{d}x^0)^2} = \frac{1}{\mathrm{d}x^0}\left(\frac{\mathrm{d}a^{(+)}}{\mathrm{d}x^0}\right) = \frac{3c_0^2}{4\pi G\rho_0}\frac{1}{2\,ch^4(v)},\end{aligned} \tag{61}$$

and similar equations for the second metric solution, so that $\delta a^{(+)}/\delta\rho_0 = \delta a^{(-)}/\delta\rho_0 = 0$ which fits our fundamental relationship (39).

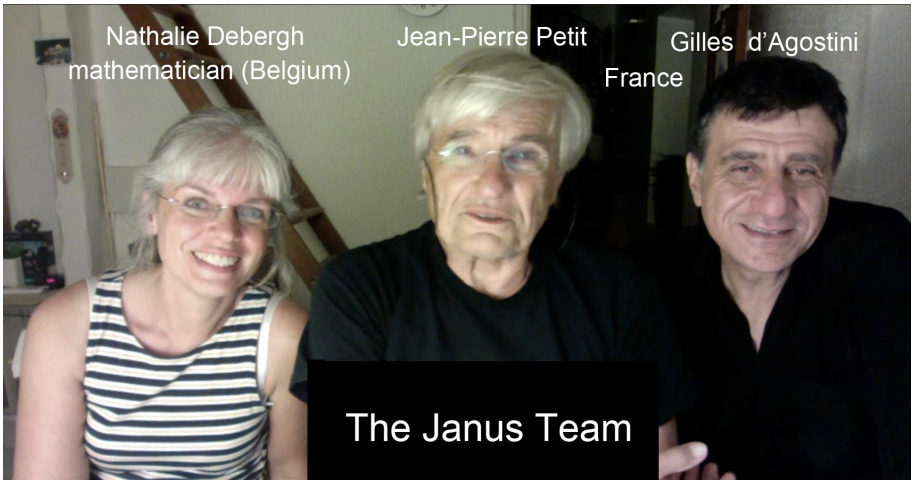
6 Conclusion

A model is never definitively fixed in time. The set of two coupled field equations first established in [9] corresponded to a first step. The present paper proposes an updated system that has been mathematically enriched to give a precise description of the matter-dominated era. In its Newtonian approximation, it provides a new insight on astrophysics, especially in galactic dynamics which no longer depends on a set of a single Vlasov equation plus Poisson but on two Vlasov equations coupled with Poisson's equation. New results in that field will be published soon.

At the present time, JCM provides:

- joint solutions $\left(g_{\mu\nu}^{(+)}, g_{\mu\nu}^{(-)}\right)$ corresponding to the functional space of Riemannian metrics of signature $(+\,-\,-\,-)$, fitting fundamental relationship $g_{\mu\nu}^{(+)} = -g_{\mu\nu}^{(-)}$.
- with stationary and spherically symmetric conditions in the vacuum.
- time dependent homogeneous and isotropic solutions.

Which cover everything that can currently be confronted with observations.



To a model already compliant with many observational data [22], a physically and mathematically coherent representation of joint geometries for positive energy and mass species, in the solar system and its neighborhood, has been added. Therefore, the Janus cosmological model agrees with classical verifications of general relativity.

By reversing this situation, considering instead a portion of space where negative mass largely dominates locally, i.e. where positive mass has been repelled away so its mass density can be taken equal to zero, we obtain the first coherent theoretical description of the *Great Repeller*, which has been exposed in [26].

When photons emitted by high redshift galaxies (z>7) cross negative mass conglomerates in the center of big cosmic voids, in the large-scale structure of the universe, negative gravitational lensing reduces their apparent magnitude, making them appear as dwarf galaxies, which is consistent with observations.

One may argue that the Janus theory exhibiting tow coupled metrics as a "natural" hypothesis with the confidence that subsequent results would eventually corroborate the postulate. However this bimetric model is formally sustained by a specific splitting of the Riemann Tensor which yields to 2nd rank tensor field equations, as shown in [27].

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